The Disintegration (or Not) of the Nonlinear Internal Tide

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LONG-TERM GOALS

This research is aimed at studying the underlying dynamics of, and identifying the conditions that control, the disintegration of the internal tide into large-amplitude internal solitary-like waves.

OBJECTIVES

The objectives are to use a combination of theoretical and numerical models to study the evolution of the internal tide and its possible disintegration into internal solitary waves. A central aspect of this work is to explore the role of rotation in the process. Rotation permits the presence of periodic, nonlinear inertia-gravity waves (i.e., the tide) that can act as attractors and arrest the steepening of the internal tide, and hence affect the production of the shorter solitary-like waves (Gerkema, 1996). In light of recent observations of strongly nonlinear internal solitary waves in the South China Sea (e.g. Duda *et al*, 2004; Ramp *et al*., 2004) and numerous other locations, an important objective is to allow for fully nonlinear waves. A further objective is to test these theories and models with observations obtained from the NLIWI South China Sea DRI in order to improve the ability to predict the arrival of large-amplitude internal solitary waves.

APPROACH

The approach combines theoretical wave evolution models and numerical solutions of these models and solutions of the full Navier-Stokes equations. The theoretical models require some simplifications including restriction to two-layer flows and one-dimensional propagation. The presence of rotation requires flow in the direction transverse to the propagation; however, variations of properties in this direction will be ignored initially. The theory is an extension of the fully-nonlinear, weakly non-hydrostatic internal wave theory of Miyata (1988) and Choi and Camassa (1999) to include rotation. In most cases these model equations will be solved numerically using modern, high-order schemes, though some analytical progress can be made. This theory will be complemented using a 2.5-dimensional Navier-Stokes numerical model that permits continuous stratification and eliminates restrictions associated with the long-wave assumption in the theory. Variable topography can be included in both the theory and the numerical models.

The theoretical and modeling results will be compared against observations (both extant and to be obtained) by David Farmer (URI) and other NILIWI investigators of the low-mode internal tide evolution across the South China Sea from just west of Luzon Strait to the Chinese shelf.

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WORK COMPLETED

This grant was awarded near the end of the current FY and work has just been initiated. However, some important components of the project have already been completed. First, the Miyata/Choi-Camassa (MCC) theory has been extended to include rotation (MCC-f) and a numerical scheme has been developed and tested. The non-hydrostatic numerical model had been previously developed by the P.I. for other research and has now been extended to include the transverse flow (2.5 dimensions) associated with rotation.

Two studies with the new MCC-f theory have been initiated. In the first, the model is used to examine the disintegration of an initially sinusoidal internal tide. In the second, the radiation decay of large-amplitude internal solitary waves with rotation has been investigated (Helfrich, 2006).

In the coming year this work will be continued and extension to continuous stratification and comparison with NLIWI observation in the South China Sea will be persued.

RESULTS

Earlier work on the role of rotation on the evolution of an internal tide had been restricted to weakly nonlinear waves (e.g., Gerkema, 1996; Holloway *et al.*, 1999). Gerkema (1996) found that with rotation the fission of the tide into solitary wave packets could be inhibited. This was due to the presence of rotational dispersion, which could balance the nonlinearity to give hydrostatic nonlinear tide (inertia-gravity) solutions that prevented further steepening of the tide, and thus the emergence of shorter solitary waves. They found that the disintegration of the tide was inhibited unless the initial tide was sufficiently nonlinear compared to the amplitude of the maximum allowable inertia-gravity wave solution, suggesting an amplitude threshold for production of solitary-like waves.

This work has been extended to the fully nonlinear regime through solutions of the MCC-f equations. This is significant because the fully nonlinearity the hydrostatic nonlinear inertia-gravity wave solutions become richer. Two examples of these calculations are shown in Figure 1. Both are for an initial tide of the same wavelength and other important parameters. The only difference is in the amplitude of the initial wave. With the smaller initial amplitude, the tide steepens, but this steepening is arrested and only rather small solitary-like waves are produced. The remaining long wave is very close to one of the hydrostatic, nonlinear tide solutions. In the second example the initial tide amplitude is larger and the disintegration of the tide into shorter internal waves is more pronounced. However, there still exists a long tide that is again very close to one of the possible hydrostatic nonlinear inertia-gravity wave solutions. This appears to be similar to Gerkema's (1996) results; however, in both examples the initial tidal amplitude was less than the maximum amplitude of the nonlinear tide solutions. The criterion for disintegration also involves the proximity of the initial condition to the underlying tide solutions, and hence illustrates the importance of the generation process. Work is continuing with a goal of understanding just how much of the energy in the initial tide can end up in the shorter solitary-like waves as a function of the independent parameters (e.g., stratification, amplitude, rotation, etc.), the time and space scales of this process, and the characteristics (e.g. amplitude and number) of the solitary-like waves.

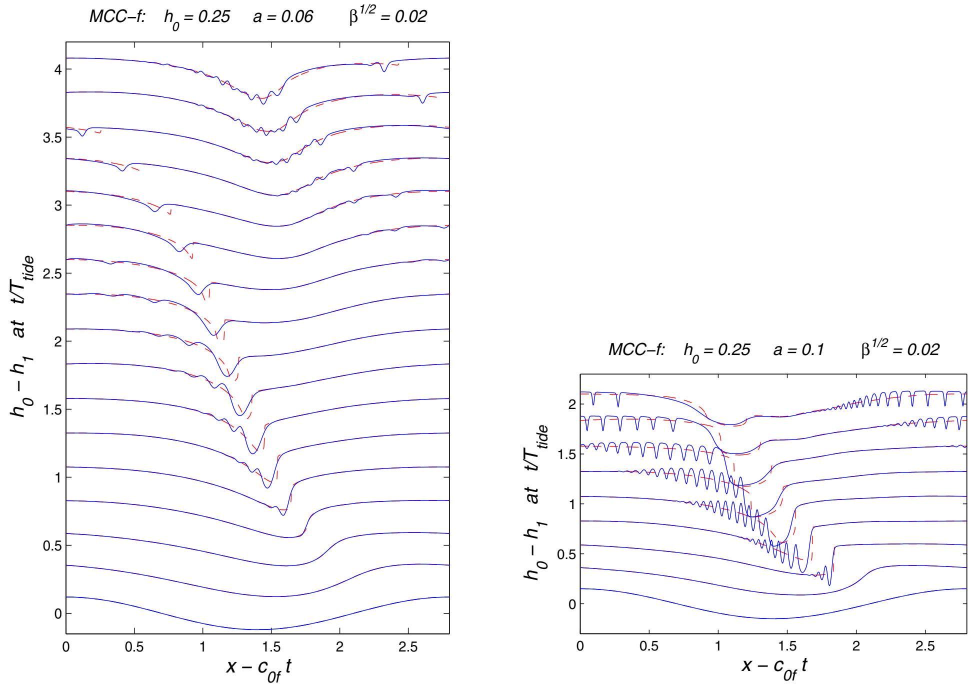

Figure 1. Evolution of an initially sinusoidal internal tide. LEFT: The figure shows the interface over several tidal periods. The solid blue line is the solution to the MCC-f equations and the dashed red line shows a solution to the hydrostatic shallow water equations (MCC-f in the hydrostatic limit). The initial tide amplitude is a=0.06. RIGHT: An identical set of computations except the initial tide amplitude a=0.1. The production of short solitary-like waves increases with amplitude, but in both cases an underlying tide remains.

From weakly nonlinear theory it is known that steadily propogating solitary wave solutions do not exist in the presence of rotation. An initial solitary wave will decay in finite time due to the radiation of longer inertia-gravity waves (Grimshaw *et al*., 1998). In a few numerical solutions the weakly nonlinear regime Grimshaw *et al*. (1998) noted that the radiation process could lead to the periodic re-emergence of a solitary wave through steepening of the radiated inertia-gravity wave.

This work has been extended into the fully-nonlinear regime through solutions of the MCC-f equations (Helfrich, 2006). The re-emergence process not only persists to large amplitude, it is enhanced. The new solutions indicate that the process is related to the generation of localized envelope, or packet, through which the solitary waves propagate, decaying at the leading edge and emerging at the trailing edge. An example of a numerical solution is shown in Figure 2. Depending on the parameters, as much as 50% of the energy in the initial wave can be retained in the packet. Current work is underway to address the dynamics of the packets.

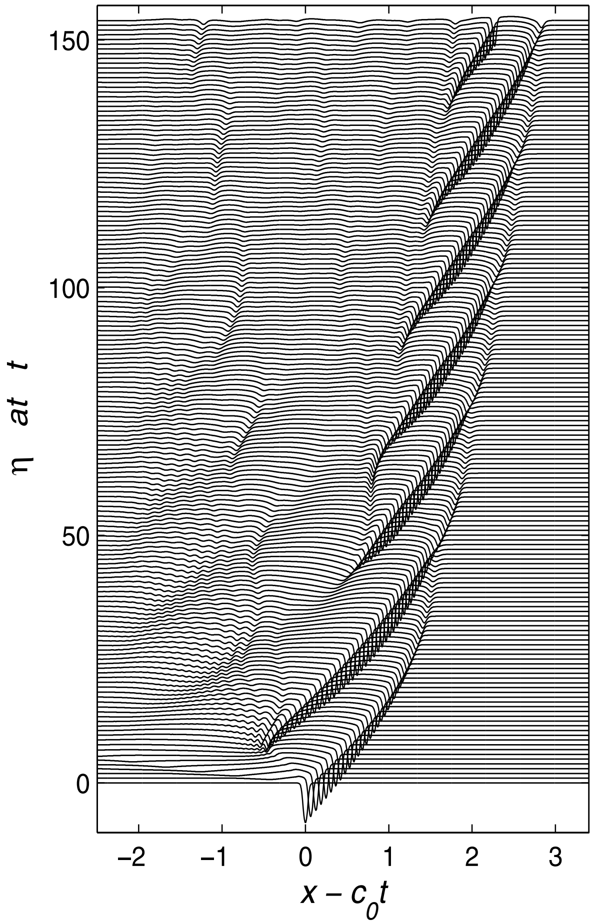

Figure 2. Decay and re-emergence of a large-amplitude internal solitary wave in the presence of rotation. The figure shows the interface in a frame moving with the linear long wave phase speed. The upper layer depth is 0.25 of the total depth and the initial wave amplitude is 0.2. The wave initially decays only to re-emerge from the steepening of the radiated inertia-gravity wave. The process repeats until a coherent packet of solitary-like waves develops. The packet is not perfectly localized, but the energy contained in the packet decays very slowly.

IMPACT/APPLICATIONS

The ubiquitous nature of large amplitude internal solitary waves in the world's coastal oceans and marginal seas is clear from observations. These waves are can have significant effects on coastal mixing through breaking as they propagate and shoal, and they may also lead to substantial horizontal mass transport. Since the waves are typically generated through the radiation of an internal tide by barotropic tidal flow over localized topography, this work will help understand what fraction of the energy put in at the tidal frequency ends up as internal solitary waves, the space and time scales for that transformation, and the characteristics of the resulting solitary-like waves.

The radiation decay and reemergence of internal solitary waves offers a possible explanation of why it is frequently difficult to track internal solitary waves. In the presence of rotation, and over the

appropriate space and time scales, the individual waves are ephemeral, though the wave energy is coherent.

This work will be directly applicable to the analysis and interpretation of the NLIWI South China Sea observations.

RELATED PROJECTS

This work is directly related to the ONR NLIWI South China Sea DRI.

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PUBLICATIONS

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